About the exceptions to Goldbach's Conjecture III

Terminological foreword

By the expression “almost all”, referred to an infinite set of integer numbers, we’ll mean “all except a finite amount”.

Hypothesis A

For almost all even numbers Npa of type 2P1, there exists at least one element of the Set G1 of the type (Px + Py) such that 2P1 = (Px + Py).

Attempt to prove Hypothesis A

Consider the Bertrand-Goldbach Theorem (statement and proof can be found in http://www.dimostriamogoldbach.it/en/proof-strategy-based-on-factorization/), on the basis of which we can state, for every even number Npa that

$$Npa = P' + Cp$$

Where P1 is between 2x = Npa and x, while Cp is a coprime number with Npa. Based on this Theorem, if there is an exception to Goldbach's Conjecture, it can be expressed as Npa = P' + Cp, where the coprime number is not a prime number with respect to Npa, but a normal odd number Nd. Taking the numbers of type 2 P1, as in any case normal even numbers Npa, we would have

$$Npa = P' + Cp$$
$$Npa = P' + Nd$$
$$2\,P1 = P' + Nd$$

The greater the number 2 P1 examined, it would not seem that this is possible, as it would be stated, for what has been said above, that the numbers 2 P1 would be exceptions to Goldbach's Conjecture, which obviously is not true: it must however be noted that the original formulation of the Bertrand-Goldbach Theorem would still remain valid, namely that 2 P1 = P' + Nd = Px + Py.
As can be seen, from this reasoning it would seem to conclude that, for every even number Npa of type 2P1, there is almost always at least one element of the G1 Set (Px + Py) such that 2P1 = (Px + Py).

Hypothesis B

In Proposition 09, where it is stated that, for sufficiently large even numbers Npa, holds that Npa = 2 P1 + P2 + P3, at least one of P1, P2 and P3 is a prime number decidable at one's own free choice, and furthermore this prime number is never the P1 that appears in 2 P1.

Comment on Hypothesis B

It is possible to prove that, in Npa = 2P1 + P2 + 3, 2P1 and P2 are actually distinct, i.e., for sufficiently large numbers Npa, the P1 appearing in 2P1, and the P2 are different prime numbers, as well as the fact that, in G2 Set, at least one of the four prime numbers involved in the formula Npa = Px + Py + Pz + Pk is a prime number decidable at one's own free choice: it is my intention, in the recent future, to disclose this passage.

Proposition 01

Extremes included, between x and 0, where x is equal to or greater than 4, there is always a number of the type 2 P1.
Every even number Npa equal to or greater than 10 can be understood as Npa = 2 P1 + Npa', where the number 2 P1 is between x and 0, with 2x = Npa.

Proof

Take the smallest 2 P1 number, i.e. the number 4, and subtract 4 from the number 10, obviously we obtain that

$$10 - 6 = 4$$

For our purposes, this can be understood as

$$10 - 6 = 4$$
$$10 - 2\ P1 = Npa$$
$$10 = 2\ P1 + Npa$$
$$Npa = 2\ P1 + Npa'$$

Clearly, this reasoning can be repeated for any number Npa, since there is always at least one number 2 P1 between x and 0, with 2x = Npa, which subtracted from the number Npa taken into consideration, gives an even number Npa'.
As it can be seen, *Extremes included, between x and 0, where x is equal to or greater than 4, there is always a number of the type 2 P1.*
Every even number Npa equal to or greater than 10 can be understood as Npa = 2 P1 + Npa', where the number 2 P1 is between x and 0, with 2x = Npa". - QED

Proposition 02

For each element of G2 Set, at least one of the two component elements belonging to G1 Set is between x and 2x, where 2x is the even number Npa equal to or greater than 8 taken into consideration.

Proof

For every even number Npa equal to or greater than 8, there exists at least one element of the G2 Set such that

$$Npa = (Px + Py) + (Pz + Pk)$$

As stated in "*On the exceptions to Goldbach's Conjecture*", since (Px + Py) and (Pz + Pk) are two elements belonging to the G1 Set, they give rise to two even numbers, and therefore

$$Npa = (Px + Py) + (Pz + Pk)$$
$$Npa = (Npa') + (Npa'')$$

This is because no number can be understood as the sum of two numbers, without at least one of the two addends not being between x and 2x with respect to the number under consideration, otherwise the sum would always be less than the number taken into consideration.
As can be seen, "*For each element of Set G2, at least one of the two component elements belonging to Set G1 is between x and 2x, where 2x is the even number Npa equal to or greater than 8 taken*

into consideration". - QED

Proposition 03

Between x and 2x, where 2x is equal to the even number Npa equal to or greater than 8 taken into consideration, there is at least one element of G1 Set.

Proof

By *Proposition 02*, for each element of G2 Set, at least one of the two component elements belonging to G1 Set is between x and 2x, where 2x is the even number Npa equal to or greater than 8 taken into consideration: moreover, for the proof by H. Helfgott, there is no even number Npa, provided it is equal to or greater than 8, which is not understandable as the sum of four prime numbers.
As it can be seen, *"between x and 2x, where 2x is equal to the even number Npa equal to or greater than 8 taken into consideration, there is at least one element of G1 Set"*. - QED

Proposition 04

Assuming Hypothesis A is true, *then between x and 2x, where 2x is equal to the even number Npa equal to or greater than 8 taken into consideration, there is almost always at least one element of the G1 Set of the (P1 + P2) kind.*

Proof

The G1 Set contains only two types of elements: even numbers of the (2 P1) kind, and even numbers of the (P1 + P2) kind. By *Proposition 03, between x and 2x, where 2x is equal to the even number Npa equal to or greater than 8 taken into consideration, there is at least one element of the Set G1*: this means that this element can either be of the 2 P1 kind, or of the (P1 + P2) kind. Therefore:

- If it is of the (P1 + P2) kind, *Proposition 04* has been proved;
- If it is of the (2 P1) kind, **assuming Hypothesis A is true**, we can state that, minus of a possible finite number of contrary cases, we almost always have a number (Px + Py) such that 2 P1 = (Px + Py).

Now, by the *Fundamental Theorem of arithmetic*, every number that is not 1 or a prime number can be expressed uniquely, regardless of the order, as a product of prime numbers: **assuming Hypothesis A is true**, if the starting number is large enough there will be at least two representations, so if one is of the (2 P1) kind, due to the uniqueness of the prime factorization, the other one must be of the (P1 + P2) kind.
As it can be seen, *"between x and 2x, where 2x is equal to the even number Npa equal to or greater than 8 taken into consideration, there is almost always at least one element of the G1 Set of (P1 + P2) kind"*. - QED

Proposition 05

All even numbers Npa equal to or greater than 12 can be understood as Npa = (P1 + P2) + (Px + Py).

Proof

For the proof of Goldbach's weak theorem by H. Helfgott, every number equal to or greater than 10 can be interpreted as Npa = (Px + Py) + (Pz + Pk).
Based on what was said in "*On the exceptions to Goldbach's Conjecture II*", for each element in the G2 Set an element of the G1 Set of the (P1 + P2) kind is a necessary but not sufficient condition, therefore we have that

$$(Npa = (Px + Py) + (Pz + Pk)) \rightarrow (Npa = (P1 + P2) + (Pz + Pk))$$

As it can be seen, *“all even numbers Npa equal to or greater than 12 are understood as Npa = (P1 + P2) + (Px + Py)”*. - QED

Proposition 06

Almost every even number Npa can be understood as Npa = 2 P1 + (Px + Py).

Proof

It is possible to estimate the numbers of 2 P1 kind between 0 and the number Npa taken into consideration, by using the Prime number theorem: in fact, to estimate how many prime numbers are between 0 and the number Npa taken into consideration, it is equivalent to estimate how many numbers of the 2 P1 kind are present between 0 and the number x = Npa/2, given that every prime number P1 between 0 and the number x can be associated with a number of the 2 P1 kind between 0 and 2x = Npa, and vice versa, every number of the 2 P1 kind between 0 and 2x can in turn be associated with a prime number P1 between 0 and the number x.
Being a one-to-one correspondence, the Set of prime numbers between 0 and x and the Set of numbers of the 2P1 kind between 0 and 2x have the same cardinality and, given that both are countable sets, due to the well-known theorem of set theory, they also have the same number of elements.
Furthermore, it is obviously possible to estimate how many exceptions to Goldbach's Conjecture there are between 0 and the even number Npa taken into consideration, through Wen Chao Lu's proof, "*Exceptional set of Goldbach number*", whose abstract summarizes the result: *“Let E(x) denote the number of even numbers not exceeding x which cannot be written as a sum of two primes. In this paper we obtain E(x) << x to the 0.879”*.
If, for example, we take the number 1000, from a calculator, 1000 / Ln (1000) is approximately equal to 144.92: this would mean that the Prime number theorem estimates that there are about one hundred and forty-four prime numbers between 0 and 1000 , and therefore as many numbers of type 2 P1 between 0 and 2000. Instead, according to Wen Chao Lu's estimate, there would be about seven hundred and ninety-seven possible exceptions to the Conjecture between 0 and 2000.
In practice, if between 0 and 2000 we have about one hundred and forty-four numbers of type 2P1, this means that 2000 can be written in about one hundred and forty-four different ways as 2000 = 2P1 + Npa, and therefore we have, in turn, about one hundred and forty-four even numbers Npa which make the expression 2000 = 2 P1 + Npa true: Wen Chao Lu's estimate tells us that there would be about seven hundred and ninety-seven possible exceptions to the Conjecture between 0 and 2000: in this case, therefore, we would have more exceptions to Goldbach's Conjecture than numbers of 2 P1 kind between 0 and the number 2x = Npa = 2000 taken into consideration.
But, if increasing the number Npa taken into consideration, the number of writings as Npa = 2 P1 + Npa were greater than the number of possible exceptions to Goldbach's Conjecture between 0 and 2x, at least one of the Npa must be a confirmation of the Conjecture, precisely because it cannot be an exception and, for the law of excluded middle, each number taken individually, provided it is equal to or greater than 4, can only be either a confirmation or an exception to the Conjecture (and, of course, since it cannot be an exception to the Conjecture, it would necessarily be a confirmation).
This is the specific case of 2000, but it is known, from results of mathematical analysis, that, the

greater the number x taken into consideration, the more the result of x/(ln x) grows faster than 2x raised to 0.879 (more in general, it would appear that x/(ln x) grows faster than any power of x with exponent less than 1).
As can be seen, *"almost every even number Npa can be understood as Npa = 2 P1 + (Px + Py)".* - QED

Comment on Proposition 06

For the continuation of the discussion, it is important to note that the reasoning of *Proposition 06* can be strengthened to demonstrate that the 2 P1 of Npa = 2 P1 + (Px + Py) is included between 2x = Npa and x.
The *Team of Letsproofgoldbach!* kindly pointed out the following:

“Basically, instead of estimating the number of primes between 0 and x with the formula x / (ln x), one would have to estimate the number of primes between x/2 and x. Broadly speaking, one could proceed as follows:

number of 2 P1 between x and 2x = number of primes between x/2 and x = (number of primes less than or equal to x) - (number of primes less than or equal to x/2) =

x / (ln x) - (x/2) / (ln x/2) =
x / (ln x) - (1/2) x / (ln x - ln 2) >= [for x large enough]
x / (ln x) - (1/2) x / (3/4 ln x) =
x / (ln x) - (2/3) x / (ln x) =
1/3 x / (ln x)

so the following arguments would still be valid: the factor 1/3 would not change the substance."

Proposition 07

The greater the even number Npa examined, the more the number of elements of G2 Set algebraically associable to an even number Npa equal to or greater than 8 tends to infinity (moreover, there are no contrary cases).

Proof

As a consequence of the methods used in the *proof* of *Proposition 06*, it is also possible to proof that the writings of a number Npa as Npa = (Px + Py) + (Pz + Pk) tend to grow indefinitely as the number Npa under consideration always becomes larger: in fact, if, as we said in *Proposition 06*, (x/ln x) grows faster than x raised to 0.879, and since there are infinitely many prime numbers between 0 and the number x, as the number x taken into consideration becomes increasingly larger, as a consequence of the fact that the difference between the number of possible 2P1s and the maximum number of exceptions tends to infinity, since we are dealing with two different orders of infinity, this means that the number of writings of a number Npa such as Npa = (Px + Py) + (Pz + Pk) tend to infinity, and more specifically, in the present case the writings of the Npa = 2 P1 + (Px + Py) kibd would tend to infinity. Finally, for the proof of Goldbach's weak theorem by H. Helfgott, there can be no contrary cases.
As it can be seen, *"The greater the even number Npa examined, the more the number of elements of Set G2 algebraically associable to an even number Npa equal to or greater than 8, tends to infinity (moreover, there are no contrary cases)"*. - QED

Proposition 08

Assuming Hypothesis A is true*, almost any even number Npa can be understood as Npa = (P1 + P2) + (Px + Py), where it is possible to state, for the element (P1 + P2), that (P1 + P2) = 2 P'.*

Proof

Keep the following assumptions in mind:

1. By *Proposition 05,* we can state that "*All even numbers Npa equal to or greater than 12 are understood as Npa = (P1 + P2) + (Px + Py)*";
2. **Assuming Hypothesis A is true**, "*for almost all even numbers Npa of type 2P1, there exists at least one element of the G1 Set of the (Px + Py) kind such that 2P1 = (Px + Py)*", which for the proof of *Proposition 05,* allows us to state that 2 P1 = (Px + Py) = (P' + P");
3. By *Proposition 01,* we can state that "*between x and 0, where x is equal to or greater than 4, there is always a number of the type 2 P1. Every even number Npa equal to or greater than 10 can be understood as Npa = 2 P1 + Npa', where the number 2 P1 is between x and 0, with 2x = Npa*", and similarly for what is said in "*On the exceptions to the Conjecture of Goldbach II*", we can state that between x and 2x = Npa there is at least one even number of type 2 P1;
4. By *Proposition 06*, we can state that "*almost every even number Npa can be understood as Npa = 2 P1 + (Px + Py)*";
5. By *Proposition 07,* we can state that *"the greater the even number Npa under consideration, the more the number of elements of the Set G2 algebraically associable to an even number Npa equal to or greater than 8 tends to infinity (moreover not there are contrary cases)"*.

Starting from these assumptions, it is possible to reason as follows.
Since it is possible to state that almost all even numbers Npa can be understood as Npa = 2 P1 + (Px + Py), and it is equally possible to state that all even numbers Npa equal to or greater than 12 can be understood as Npa = (P1 + P2) + (Px + Py), these two methods of writing must inevitably meet almost always, **if Hypothesis A is assumed to be true**, because it can almost always be stated that a number 2 P1 can be understood as 2 P1 = P' + P", and because between 2x = Npa and x there is always at least one even number of type 2 P1, and the same happens between x and 0, with 2x always equal to Npa.
As we see, *"almost any even number Npa can be understood as Npa = (P1 + P2) + (Px + Py), where the element it is possible to state, for the element (P1 + P2), that (P1 + P2) = 2 P' "*. - QED

Proposition 09

Assuming Hypothesis A is true, *almost any even number Npa can be understood as Npa = (2 P1) + (P2 + P3), or, which is the same, almost any even number Npa of the form Npa = (P1 + P2) + (P3 + P4) finds an equivalent as Npa = (2 P1) + (P2 + P3).*

Proof

By *Proposition 08* and its *Corollary*, *"almost all even numbers Npa can be understood as Npa = 2 P1 + (Px + Py)"*, where the number 2 P1 is included between 2x = Npa and x: therefore, the other element of the G1 Set, i.e. (Px + Py), either it is also between 2x = Npa and x, or between x and 0, with always 2x = Npa.
In the first case, i.e. the one in which the element (Px + Py) is located between 2x = Npa and x, this situation is possible only when (Px + Py) is equal, numerically speaking, to the number 2 P1: in fact, if if this were not the case, the number resulting from the sum would be greater than the number Npa taken into consideration (therefore it would only be a matter of cases such as 20 = 10 + 10 = (5 +5) + (7 + 3) etc).

But as we said in the *Proof* of *Proposition 04*, a number 2 P1 admits other writings as the sum of two prime numbers only in the form (P' + P"), and obviously P1 of 2 P1, P' and P" are necessarily distinct from each other.
In the second case, i.e. the one in which (Px + Py) lies between x and 0, with always 2x = Npa, since, as stated in *"On the exceptions to Goldbach's conjecture"*, (Px + Py) is an element of G1 Set, it can be either of (P' + P") kind or of (2 P') kind:

- If the element is of the (P' + P") kind, since this element of the G1 set is between x and 0 with respect to Npa = 2x, P' and P" are necessarily distinct from the P1 of 2 P1;
- If the element is of (2 P') kind, **assuming Hypothesis A is true,** it can almost always be understood as (P' + P"): and being also included between x and 0, with 2x = Npa, even in this case P' and P" are necessarily distinct from the P1 of 2 P1.

As we see, *"**assuming Hypothesis A is true,** almost any even number Npa can be understood as Npa = (2 P1) + (P2 + P3), or, which is the same thing, almost any even number Npa of the type Npa = (P1 + P2) + (P3 + P4) has an equivalent as Npa = (2 P1) + (P2 + P3)"*. - QED

Proposition 10

Assuming that Hypothesis B is true, *almost any odd number Nd can be understood as Nd = 2 P1 + P2.*

Proof

Assuming that Hypothesis B is true, any even number Npa can be written as Npa = 2 P1 + P2 + 3.
As already done in *"On the Exceptions to Goldbach's Conjecture II"*, if 3 is subtracted from the even number Npa large enough to say that Npa = 2 P1 + P2 + 3, one obtains almost all the corresponding odd numbers. Indeed:

Npa = 2 P1 + P2 + 3
Npa - 3 = 2 P1 + P2
Npa – 3 = 2 P1 + P2
Nd = 2 P1 + P2

In fact, if 30 were the even number Npa large enough to state that almost all even numbers Npa can be written as Npa = 2 P1 + P2 + 3, subtracting 3 from 30 we get 27, subtracting 3 from 32 we get 29, subtracting 3 from 34 we get 31, and so on, getting almost all odd numbers Nd.
As we can see, *"**assuming that Hypothesis B is true,** almost any odd number Nd can be understood as Nd = 2 P1 + P2"*. - QED

Proposition 11

Assuming that Hypothesis B is true, *the larger the even number Npa taken into consideration, the more almost any even number Npa can be understood as Npa = P1 + (2 P2 + 1).*

Proof

By *Proposition 10*, *"almost any odd number Nd can be understood as Nd = 2 P1 + P2"*: therefore, passing to the even numbers by adding 1, instead of 3, we have

Nd = 2 P1 + P2

$$Nd + 1 = 2\ P1 + P2 + 1$$
$$Npa = 2\ P1 + P2 + 1$$
$$Npa = P1 + (2\ P2 + 1)$$

As can be seen, *"**assuming that Hypothesis B is true,** the greater the even number Npa under consideration, the more almost any even number Npa can be understood as Npa = P1 + (2 P2 + 1)."* - QED

Proposition 12

Assuming that Hypothesis B is true, *almost every exception to Goldbach's Conjecture can be expressed as Npa = P1 + (2 P2 + 1), and almost every even number Npa can be understood as either Npa = Px + Py, or Npa = P1 + (2 P2 + 1).*

Proof

Every even number Npa equal to or greater than 4 is either a confirmation of Goldbach's Conjecture, and then this even number Npa can be understood as Npa = (Px + Py), or it is an exception to Goldbach's Conjecture: the greater the even number Npa considered, if at least one exception to Goldbach's Conjecture exists, than, by *Proposition 11*, this number can be understood as Npa = P1 + (2 P1 + 1).
Thus, for sufficiently large Npa, any even number Npa can be expressed either as Npa = (Px + Py), or as Npa = P1 + (2 P2 + 1).
As we see, *"assuming that Hypothesis B is true, almost every exception to Goldbach's Conjecture can be expressed as Npa = P1 + (2 P2 + 1), and almost every even number Npa can be understood as either Npa = Px + Py, or as Npa = P1 + (2 P2 + 1)"*. - QED

Comment on Proposition 12

Proposition 12 can be considered one of the most important results obtained by these attempts: as we can see, it is more powerful than the Yamada-Chen Theorem. Nonetheless, some reasons for dissatisfaction could emerge, on a general level.

A first reason for dissatisfaction is that the formula Npa = P1 + (2 P2 + 1), being independent of Goldbach's Conjecture, which is positive if one wants to try to identify possible exceptions, nevertheless can create problems if one wants to try to pursue a demonstration from it. In this regard, it should be noted that this problem can be solved by taking the formula Npa = Px + (P1 + P2 + 1), clearly derived from Nd = Px + (P1 + P2), proved in *"On the exceptions to Goldbach's conjecture II"*. Theoretically speaking, if Goldbach's Conjecture is true, (P1 + P2 + 1) can always be a prime number, and this for any even number.
A second reason for dissatisfaction is due to the fact that Npa = P1 + (2 P2 + 1) is expressed through additive language, instead of multiplicative, as Yamada-Chen's theorem does: unfortunately, current techniques are largely implemented on a multiplicative, i.e. related to the Fundamental Theorem of arithmetic, as it is proved, while using an additive basis, based on Goldbach's Conjecture, when, apart from odd numbers, it has not yet been proved, does not seem to offer a useful margin of work.

Proposition 13

Assuming that Hypothesis B true, Proposition 12 is independent of Goldbach's Conjecture: that is, there exists at least one even number Npa such that, in Npa = P1 + (2 P2 + 1), the number (2 P2 + 1) is a normal odd number Nd, and not a prime number.

Proof

This is an empirical check of the formula Npa = P1 + (2 P2 + 1) on even numbers from 8 up to 100. This control was carried out, in regular cases, with the tool found at (http://www.dimostriamogoldbach.it/en/goldbach-pairs-viewer/), while in non-regular cases, marked in red, it was used a list of prime numbers, found in (https://en.wikipedia.org/wiki/List_of_prime_numbers), and then checked with a normal calculator:

8 = 3 + (2 + 2 + 1)
10 = 5 + (2 + 2 + 1)
12 = 7 + (2 + 2 +1)
14 = 3 + (5 + 5 + 1)
16 = 11 + (2 + 2 + 1)
18 = 11 + (3 + 3 + 1)
20 = 13 + (3 + 3 + 1)
22 = 17 + (2 + 2 + 1)
24 = 17 + (3 + 3 + 1)
26 = 19 + (3 + 3 + 1)
28 = 17 + (5 + 5 + 1)
30 = 23 + (3 + 3 + 1)
32 = 5 + (13 + 13 + 1)
34 = 23 + (5 + 5 + 1)
36 = 29 + (3 + 3 + 1)
38 = 31 + (3 + 3 + 1)
40 = 29 + (5 + 5 + 1)
42 = 37 + (2 + 2 + 1)
44 = 37 + (3 + 3 + 1)
46 = 41 + (2 + 2 + 1)
48 = 41 + (3 + 3 + 1)
50 = 43 + (3 + 3 + 1)
52 = 41 + (5 + 5 + 1)
54 = 43 + (5 + 5 + 1)
56 = 17 + (19 + 19 + 1)
58 = 47 + (5 + 5 + 1)
60 = 53 + (3 + 3 + 1)
62 = 47 + (7 + 7 + 1)
64 = 59 + (2 + 2 + 1)
66 = 61 + (2 + 2 + 1)
68 = 61 + (3 + 3 + 1)
70 = 59 + (5 + 5 + 1)
72 = 67 + (2 + 2 + 1)
74 = 67 + (3 + 3 + 1)
76 = 71 + (2 + 2 + 1)
78 = 73 + (2 + 2 + 1)
80 = 73 + (3 + 3 + 1)
82 = 71 + (5 + 5 + 1)
84 = 79 + (2 + 2 + 1)
86 = 79 + (3 + 3 + 1)
88 = 83 + (2 + 2 + 1)
90 = 83 + (3 + 3 + 1)
92 = 17 + (37 + 37 + 1)
94 = 89 + (2 + 2 + 1)

96 = 89 + (3 + 3 + 1)
98 = 11 + (43 + 43 + 1)
100 = 89 + (5 + 5 + 1)

As can be seen, *“**assuming that Hypothesis B is true,** Proposition 12 is independent of Goldbach's Conjecture, i.e. there exists at least one even number Npa such that, in Npa = P1 + (2 P2 + 1), the number (2 P2 + 1) is an odd number Nd, and not a prime number”*. - QED

Final considerations

What does it mean to "reduce" even numbers of a certain shape?

In previous talks here there has been a lot of discussion about "reducing" a certain type of even numbers Npa having a certain shape, e.g. Npa = 4 P1, to other even numbers Npa having another form, such as e.g. a Npa = Px + (P1 + P2 + P3). More rigorously speaking, what does this largely intuitive expression mean?
When we speak of "form", obviously we mean a specific formula algebraically understood, e.g., Npa = 4 P1 is a different "form" from Npa = 2 P1 + 2 P2 because, from an algebraic point of view, it is of different situations.
But what does it mean, more strictly, that a shape, e.g. Npa = 4 P1, is it reduced to Npa = Px + (P1 + P2 + P3 + P4)?
Still using the above example, it means that, if we take a determined even number of the type Npa = 4 P1, in the G2 Set we always find at least one alternative form for the same even number between Npa = P1 + P2 + P3 + P4 or Npa = 2 P1 + P2 + P3. Quite satisfactory explanation, but which can be improved and anchored to an already known specific set operation, i.e. the set difference operation.
Basically, we can understand the Set G2 composed of the union (understood as a set) of subsets each containing all the infinite elements for each possible form in which, expressing all the prime numbers involved in a distinct manner, it is possible to understand an even number as the sum of four prime numbers. It follows that we can list the following subsets that make up Set G2:

- Npa = 4 P1, i.e. all the elements of G2 that have the form 4 P1, which we will call G2(A);
- Npa = 2 P1 + 2 P2, i.e. all the elements of G2 that have the form 2 P1 + 2 P2, which we will call G2(B);
- Npa = 3 P1 + P2, that is, all the elements of G2 that have the form 3 P1 + P2, which we will call G2(C);
- Npa = P1 + P2 + P3 + P4, i.e. all the elements of G2 that have the form P1 + P2 + P3 + P4, which we will call G2(D);
- Npa = 2 P1 + P2 + P3, that is, all the elements of G2 that have the form 2 P1 + P2 + P3, which we will call G2(E);

Theoretically speaking, considering the general trend of the discussion, it would be necessary to add a particular subset, i.e. the one composed of the numbers Npa = Px + Py + Pz + 3, but for simplicity it is considered as dispersed in various capacities among those previously listed (this will be discussed later).

If we use the set union operation with these 5 subsets, we get the G2 Set, i.e.

G2 = G2(A) + G2(B) + G2(C) + G2(D) + G2(E)

In this sense, the operation of "reduction", which has been spoken of so many times, consists in applying the operation of set difference to the G2 Set, thus defined.
This, moreover, is the only way to make plausible one of the propositions demonstrated in *"On the exceptions to Goldbach's Conjecture II"*, where it is stated that each element of G2 Set has as a necessary but not sufficient element an element of G1 Set of the (P1 + P2) kind: as we can see, strictly speaking this proposition would be false. The G2 Set also contains elements of the type 4 P1, which clearly falsify this statement: it is true only if the G2 Set is restructured with set difference strokes every time a reduction is made (and in fact, under these conditions, the proposition is true).

Exactly what is the G2 Set made up of?

The kind reader will probably have noticed several times the somewhat amusing turn of phrase when speaking about G2 Set: several times, even with the *Team of Letsproofgoldbach!*, the question has emerged, and it deserves to dwell on it.

E. g., looking at what was said above, : “Npa = 4 P1, that is, all the elements of the G2 Set which have the form 4 P1”, is a very silly way of expressing something. Then exactly of what is the G2 Set made up of? Reading expressions like these, which are quite common in these papers, the question arises spontaneously: and the answer is that, somewhat counterintuitively, mostly the G2 Set is not made up of even numbers. In reality, more or less surreptitiously, the set of numbers equal to or greater than 8, or 12 (depending on the case, it has already been discussed), is used to give a numerical anchoring to the elements of G2 Set, or rather, to the various and different elements of the G2 Set which are equal to the same even number.

That the strictly numerical aspect has actually been put into the background, as can be seen from a similar case:

28 = (7 + 7) + (11 + 3), and of course (7 + 7) = (11 + 3) = 14.

Or think about these other examples

20 = (5 + 5) + (5 + 5);
20 = (5 + 5) + (7 + 3), and of course (5 + 5) = (7 + 3) = 10.

Most likely, this is the partly involuntary result of how the reasoning has been set up : in a nutshell, the algebraic aspect has surpassed the fully numerical one.

The only explanation why cases like 20 = (5 + 5) + (5 + 5), i.e. 20 = 4 P1, and 20 = (5 + 5) + (7 + 3), i.e. 20 = 2 P1 + P2 + P3 can be distinct and different elements in the G2 Set, is that, in reality, the G2 Set is composed of the way in which four prime numbers can be added together. From an algebraic point of view, there is a lot of difference, for example, if two equal prime numbers are added together (P1 + P1 = 2 P1), or if two different prime numbers are added together (P1 + P2): unfortunately, very often, numerically this algebraic subtlety is non-existent. This is already well known in the G1 Set: just think of 7 + 7 = 14, and 11 + 3 = 14, which, although they are algebraically different operations, do not give rise to a real numerical difference, and with G2 Set, as we have seen, things are only more amplified (the example with the number 20 makes it quite clear of the level that this situation can reach).

Obviously, for now, there is little to be done about this: although sometimes it helps, and although other times it does not, this kind of approach to whole G2 Set is the basis for how this reasoning has been developed.

In this sense, by admitting that the algebraic aspect has at least partially taken the upper hand, it is explained why the subset of the G2 Set, composed of the numbers Npa = Px + Py + Pz + 3, is little thematized, despite being very important, in partly due to obvious explanatory difficulties (an attempt has been made to contain the level of abstruseness in reasoning), and partly because its hybrid nature makes it difficult to treat from a perspective such as the one adopted these papers.

The *Team of Letsproofgoldbach!* has kindly offered a more exact formalization of G2 Set, for those who have or feel the need for it:

“We can define the set G2'(i), for an even integer i >= 8, as the set of all sets of four prime numbers {p1, p2, p3, p4} such that p1 + p2 + p3 + p4 = i.
For example:
G2'(8) = {{2, 2, 2, 2}}
G2'(10) = {{2, 2, 3, 3}}

G2'(12) = {{2, 2, 3, 5}, {3, 3, 3, 3}}
G2'(14) = {{2, 2, 5, 5}, {2, 2, 3, 7}, {3, 3, 3, 5}}
G2'(16) = {{2, 2, 5, 7}, {3, 3, 3, 7}, {3, 3, 5, 5}}
and so on.
According to H. Helfgott, G2'(i) is not empty, for all i.
Note that, since the elements of G2'(i) are sets, the elements have no particular order, for example in G2'(12): {2, 2, 3, 5} = {2, 2, 5, 3}.
Now we can define G2(i) as the set of possible sets of the type {{p1, p2}, {p3, p4}} such that {p1, p2, p3, p4} belongs to G2'(i), p1 + p2 is even and p3 + p4 is even.
For example:
G2(8) = { ({2, 2}, {2, 2}) }
G2(10) = { {{2, 2}, {3, 3}} }
G2(12) = { {{2, 2}, {3, 5}}, {{3, 3}, {3, 3}} }
G2(14) = { {{2, 2}, {5, 5}}, {{2, 2}, {3, 7}}, {{3, 3}, {3, 5}} }
G2(16) = { {{2, 2}, {5, 7}}, {{3, 3}, {3, 7}}, {{3, 3}, {5, 5}}, {{ 3, 5}, {3, 5}} } (note that this has one more element than G2'(16))
and so on.
The element {{p1, p2}, {p3, p4}} can be represented, if desired, as (p1 + p2) + (p3 + p4); however, the set notation is more precise and allows to eliminate the sums obtained both through the exchange of (p1+ p2) with (p3 + p4), and through the exchange of p1 with p2 and p3 with p4.
Thus, for example, the statement "12 = 2P1 + (Px + Py)" would become: "there exist Px and Py such that {{P1, P1}, {Px, Py}} belongs to G2(12)" or "G2(12) contains an element of the type {{P1, P1}, {Px, Py}}". It may seem more complex, but this formalization can help clarify some concepts that might otherwise be ambiguous.
Finally, of course, the "total" set G2 can be defined as the union of all sets G2(i) for all even i >= 8.”